# 3D Fluid Flow Reconstruction Using Compact Light Field PIV Supplementary Material


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In this supplementary document, we describe our optimization procedure to solve the 3D velocity field $\mathbf{u} = [u, v, w]$ using Eq. 7.

We solve the optimization problem via variational method. Specifically, we take the derivative of $\mathbf{u}$ in order to solve the problem with linear equations. Given an initial $\mathbf{u}$, our goal is to solve the incremental amount $\Delta\mathbf{u}$ in each iteration. Our objective function $E_{total}(\mathbf{u})$ is then become $E_{total}(\Delta\mathbf{u})$. We set out to find $\Delta\mathbf{u} = [\Delta u, \Delta v, \Delta w]$ that minimizes the first-order gradients $\frac{\partial E}{\partial u}, \frac{\partial E}{\partial v}$, and $\frac{\partial E}{\partial w}$.

To form a linear system, we unroll $u$, $v$, $w$, $u_{sol}$, $v_{sol}$, $w_{sol}$, $\Delta u$, $\Delta v$, and $\Delta w$ into column vectors. We use capitalized symbols $U$, $V$, $W$, $U_{sol}$, $V_{sol}$, $W_{sol}$, $\Delta U$, $\Delta V$, and $\Delta W$ to represent these column vectors. Let $\Theta_{x,y,z}$ be the partial derivatives of $\Theta_2(p + \mathbf{u})$ and $\Theta_t = \Theta_2(p + \mathbf{u}) - \Theta_1$. Let the $\mathbf{\Theta}_x = diag(\Theta_x)$, $\mathbf{\Theta}_y = diag(\Theta_y)$, $\mathbf{\Theta}_z = diag(\Theta_z)$ and $\mathbf{\Theta}_t = diag(\Theta_t)$ be the diagonal matrices where the diagonal values are vectorized volumes of $\Theta_x$, $\Theta_y$, $\Theta_z$, and $\Theta_t$. We can then form the linear system $\mathcal{A}\mathbf{x} = \tilde{b}$ to solve $\Delta\mathbf{u}$, where $\mathbf{x} = \Delta\mathbf{u}^T$.

$$\mathcal{A} = \begin{bmatrix} \mathbf{\Theta}_x^2 + \mathcal{F} & \mathbf{\Theta}_x\mathbf{\Theta}_y & \mathbf{\Theta}_x\mathbf{\Theta}_z \\ \mathbf{\Theta}_x\mathbf{\Theta}_y & \mathbf{\Theta}_y^2 + \mathcal{F} & \mathbf{\Theta}_y\mathbf{\Theta}_z \\ \mathbf{\Theta}_x\mathbf{\Theta}_z & \mathbf{\Theta}_y\mathbf{\Theta}_z & \mathbf{\Theta}_z^2 + \mathcal{F} \end{bmatrix} \tag{1}$$

$$\tilde{b} = \begin{bmatrix} -(\mathbf{\Theta}_x\mathbf{\Theta}_t + \mathcal{F}U - \lambda_2 F U_c - \lambda_3 F_{\mathbb{I}} U_{sol}) \\ -(\mathbf{\Theta}_y\mathbf{\Theta}_t + \mathcal{F}V - \lambda_2 F V_c - \lambda_3 F_{\mathbb{I}} V_{sol}) \\ -(\mathbf{\Theta}_z\mathbf{\Theta}_t + \mathcal{F}W - \lambda_2 F W_c - \lambda_3 F_{\mathbb{I}} W_{sol}) \end{bmatrix} \tag{2}$$

In our formulations, $\mathcal{F} = \lambda_1 L + \lambda_2 F + \lambda_3 F_{\mathbb{I}}$, where $F_{\mathbb{I}}$ represents identity matrix and $L$ is the Laplacian filter defined as $L = D_x^T D_x + D_y^T D_y + D_z^T D_z$. $U_c$, $V_c$ and $W_c$ are the matrices computed by the sparse correspondence described in Section 3.2.

Furthermore, in order to improve the accuracy, we perform the optimization with a multi-layer coarse-to-fine scheme that uses the Gaussian pyramid. In our implementation, the initial value of $\mathbf{u}$ is set to zero. We then update $\mathbf{u}$ at each level by propagating $\Delta\mathbf{u}$ from coarse to fine.