DOPbox
Demonstration of the Chebyshev Polynomials

Matthew Harker and Paul O'Leary
Institute for Automation
University of Leoben
A-8700 Leoben, Austria
URL: automation.unileoben.ac.at

Original: January 20, 2013

Last Modified: April 11, 2013

Contents

1 Setting things up

Clear up the workspace

```
clear;
close all;
setUpGraphics;
%
% Define the number of points and the number of basis functions
%
noPts = 100;
noBfs = 5;
```

2 Generate the nodes and basis functions

Generate the Chebyshev nodes

```
x = dopNodes( noPts, 'cheby');
%
% Setup the first two basis functions
%
t0 = ones( noPts, 1 );
t1 = x;
%
% Use the Chenbshev recurrence relationship
%
T = zeros( noPts, noBfs );
T(:,1) = t0;
T(:,2) = t1;
for k=3:noBfs
   T(:,k) = 2 * x .* T(:,k-1) - T(:,k-2);
end;
```

3 Determine the quality of the basis functions

The discrete basis functions should form an orthogonal (but not orthonormal) set of basis functions. The Gram matrix is defined as $\boldsymbol{G} \triangleq \boldsymbol{T}^{\mathrm{T}} \boldsymbol{T}$.

```
G = T' * T;
%
% Removing the diagonal elements should yield a matrix of
% zeros.
%
Gperp = diag(diag(G)) - G;
%
% The Frobenius norm of Gperp is now used as a measure of quality for
        the
```

```
% basis functions.
%
e = (norm( Gperp, 'fro'));
```

and the $n_d = -log_{10}$ as an estimate for the number of available digits.

```
nd = - log10( e );
```

4 Plot the basis functions

Plot the Chebyshev basis functions

```
figure
plot( x, t1, 'k' );
hold on;
for k=2:noBfs
   plot( x, T(:,k), 'k' );
end;
grid on;
xlabel('Support');
ylabel('Value');
title(['The first n = ',int2str( noBfs ),' Chebyshev basis functions ( no digits n_d = ',num2str(nd),')']);
```

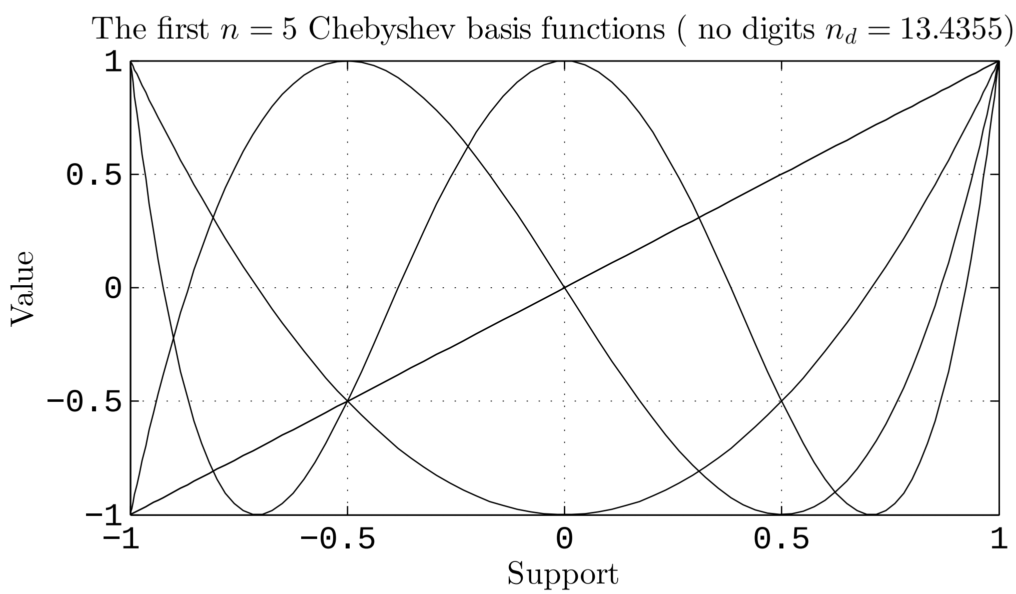

Figure 1: Some Chebyshev polynomials.

5 Plot the matrix of errors

Plot the error matrix.

```
fig2 = figure;
imagesc( Gperp );
colorbar;
axis image;
xlabel('Basis function no.');
ylabel('Basis function no.');
```

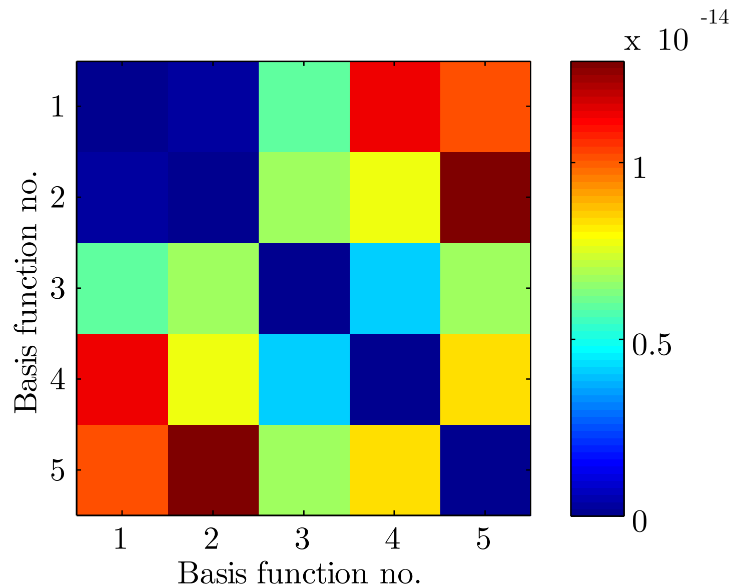

Figure 2: Projection of the basis functions onto their orthogonal complement.